RGB: the Really Getting Bayes game

A. Critch, 2012/03/24 (page 1 of 2)

Latest version: http://math.berkeley.edu/~critch/mphd/rgb.pdf

Tips for teaching the game: http://math.berkeley.edu/~critch/mphd/teaching-tips.html

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Description

Players: Two people named The Agent and Reality.

Purpose: To physically practice thinking through what Bayes' Rule says about how to update everyday beliefs, in real time, rather than in an abstract or purely academic setting.

Prerequisite: A basic understanding of Bayes' Theorem and/or Bayes' Rule. See, e.g., http://betterexplained.com/articles/an-intuitive-and-short-explanation-of-bayes-theorem/, or http://en.wikipedia.org/wiki/Bayes'_rule#Single_event.

Terms: In RGB, being "**RGB-colored**" means having at least one of the colors red, green, or blue visible on its exterior. Over-analyzing what this means it not allowed: uncertainty about what people mean by "red" is part of the game, and life!

Instructions (basic version)

Throughout the game, The Agent is not allowed to directly see what Reality is doing. He closes his eyes or looks away, and has to infer her choices from clues that she gives him.

Step 1: (setup) Reality chooses a secret object from nearby that is easy to lift, for example, a <u>binder</u>. She then chooses another type of object which may or may not be present, for example, a hair clip, and chooses an order for announcing them.

Reality says: "I'm about to give you sense data from an object that's either (e.g.) a <u>binder</u> or a <u>hair clip</u>. Be curious which one it is!"

Step 2: (prior experience) Reality touches The Agent lightly with the object in some way that gives The Agent a tiny amount of sense data, hopefully not completely revealing the identity of the object. (Reality can give The Agent instructions to facilitate this.)

Step 3: (prior odds) Reality asks: "What subjective odds do you assign to the object being (e.g.) a binder vs a hair clip?"

The Agent answers: "I give X:Y odds of it being (e.g.) a binder vs a hair clip."

(Subjective odds are a ratio of subjective probabilities, P(binder)/P(hair clip), which is often immensely easier to compute or intuit than than the normalized probabilities themselves.)

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Step 4: (likelihood comparison) Now The Agent effectively makes a comparison between **P**(the object is RGB-colored | it's a binder) and

P(the object is RGB-colored | it's a <u>hair clip</u>): Reality asks: "In which situation would you be more confident that the object is RGB-colored: (e.g.) if it's a <u>binder</u>, or if it's a <u>hair clip</u>?"

The Agent is allowed to think about this for a moment.

The Agent answers: "I'd be more confident that it is RGB-colored (e.g.) if it's a <u>binder</u> than if it's a <u>hair clip</u>."

If the agent deeply considers these alternatives to be exactly equally likely, new objects need to be chosen, the game starts over, and everyone is a little annoyed at The Agent.

Step 5: (new information) Reality informs The Agent whether the object is RGB-colored or not. She *does not reveal* which *particular* colors are on the object.

Step 6: (posterior odds) Reality asks: "What subjective odds do you **now** assign to the object being (e.g.) a <u>binder</u> vs a <u>hair clip</u>?"

The Agent answers: "I **now** give Z:W odds of it being (e.g.) a <u>binder</u> vs a <u>hair clip."</u>

Reality asks: "Has your confidence in the hypothesis that object is (e.g.) a <u>binder</u> **gone up** or **gone down** from your prior?"

The Agent answers: "up" or "down".

Step 7: (check with Bayes) The players check to see if the direction of change in Step 6 is consistent with Bayes' Theorem applied to Steps 4 (likelihood comparison) and Step 5 (new information). In particular, note the answer "didn't change" is not compatible with Step 4!

If, in everyday life, one finds that the belief update is *not consistent with Bayes' Rule,* something odd is going on with the beliefs involved, and they should be thought through more carefully if the issue is important.

Full version

To internalize the full implications of Bayes' Theorem, Step 4 should be replaced with estimating two odds ratios, one of which should be multiplied by the prior odds to get the posterior odds, depending on the new information. To learn more, start by checking out the Wikipedia articles on "Bayes' rule", "Bayes' theorem", "Bayesian updating", or this tutorial: http://betterexplained.com/articles/an-intuitive-and-short-explanation-of-bayes-theorem/