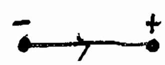
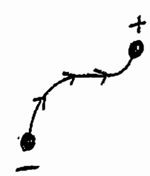
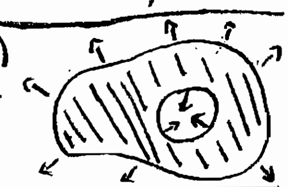
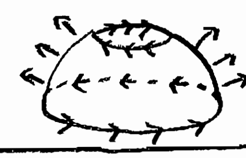
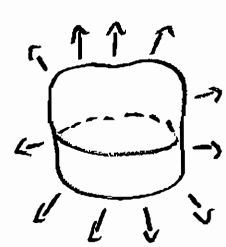
Summary / Comparison of Integral Theorems [Andrew Critch, Math 53]

Say $I=[a;b]\subseteq\mathbb{R}^1$ is an interval. Writing $\partial I = @b - @a$ to mean that $\Big|_{\partial I} f$ denotes $f(b)-f(a)$, and similarly $\partial \underset{\to}{C} = @\text{end}\,\underset{\to}{C} - @\text{beg}\,\underset{\to}{C}$ for an oriented curve, we can see some striking similarities:

		Boundary	Region
FTC on intervals	$\Big\|_{\partial I} f = \int_I f'dx = \int_I df$	oriented points in $\mathbb{R}^1$	(oriented) curve in $\mathbb{R}^1$ 
FTC on oriented curves	$\Big\|_{\partial \underset{\to}{C}} f = \int_{\underset{\to}{C}} \nabla f \cdot = \int_{\underset{\to}{C}} df$	oriented points in $\mathbb{R}^n$	oriented curve in $\mathbb{R}^n$ 
Green's Scurl Thm	$\int_{\partial D} \mathbb{F}\cdot = \iint_D \mathrm{scurl}(\mathbb{F})\,dA$	oriented curves in $\mathbb{R}^2$	(oriented) surface in $\mathbb{R}^2$ 
Green's div Thm	$\int_{\partial D} \star\mathbb{F}\cdot = \iint_D \mathrm{div}(\mathbb{F})\,dA$	oriented curves in $\mathbb{R}^2$	(oriented) surface in $\mathbb{R}^2$ 
Stokes' Curl Thm	$\int_{\partial \underset{\to}{S}} \mathbb{F}\cdot = \iint_{\underset{\to}{S}} \star\mathrm{curl}(\mathbb{F})\cdot$	oriented curves in $\mathbb{R}^3$	oriented surface in $\mathbb{R}^3$ 
Gauss' div Thm	$\iint_{\partial E} \star\mathbb{F}\cdot = \iiint_E \mathrm{div}(\mathbb{F})\,dV$	oriented surfaces in $\mathbb{R}^3$	(oriented) solid in $\mathbb{R}^3$ 

In fact, the last four theorems are even more similar than they look!

For those interested, using the wedge product "$\wedge$" and the exterior derivative "d", in $\mathbb{R}^2$ it is true that $d(\mathbb{F}\cdot) = \text{scurl}(\mathbb{F})\,dA$ and $d(\star\mathbb{F}\cdot) = \text{div}(\mathbb{F})\,dA$, and in $\mathbb{R}^3$, $d(\mathbb{F}\cdot) = \star\,\text{curl}(\mathbb{F}\cdot)$ and $d(\star\mathbb{F}\cdot) = \text{div}(\mathbb{F})\,dV$,

So we have:

FTC on intervals	$\Big\vert_{\partial I} f = \int_I df$	ooo
FTC on oriented curves	$\Big\vert_{\partial \underset{\to}{C}} f = \int_{\underset{\to}{C}} df$	ooo
Green's Scurl Thm	$\int_{\partial D} \mathbb{F}\cdot = \iint_D d(\mathbb{F}\cdot)$	ooo
Green's div Thm	$\int_{\partial D} \star\mathbb{F}\cdot = \iint_D d(\star\mathbb{F}\cdot)$	ooo
Stokes' Curl Thm	$\int_{\partial \underset{\to}{S}} \mathbb{F}\cdot = \iint_{\underset{\to}{S}} d(\mathbb{F}\cdot)$	ooo
Gauss' div Thm	$\iint_{\partial E} \star\mathbb{F}\cdot = \iiint_E d(\star\mathbb{F}\cdot)$	ooo
Stokes general Thm	$\int_{\partial M} \omega = \int_M d\omega$	oriented manifold in any other manifold!