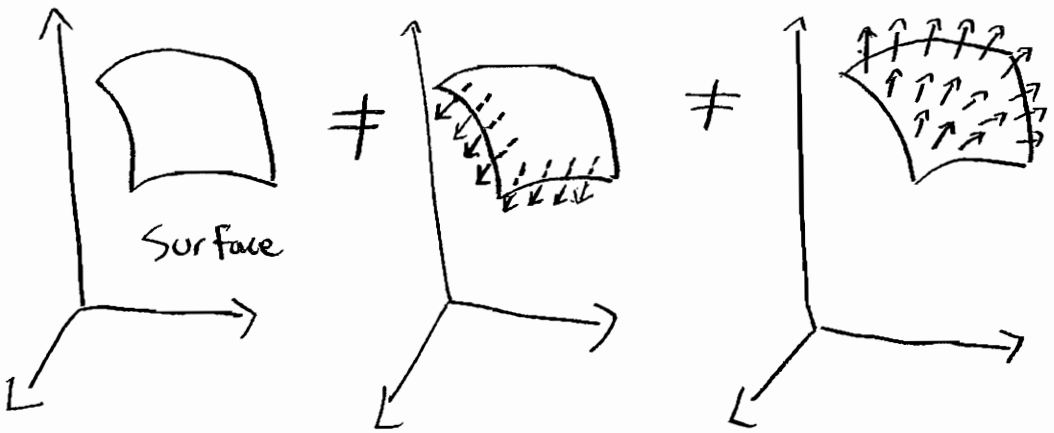
Oriented surfaces [Andrew Critch, Math 53]

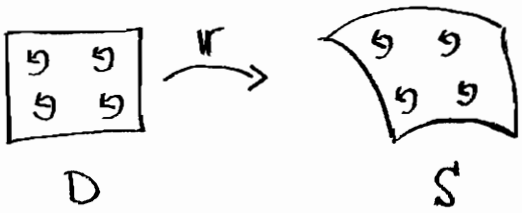
An oriented surface $\vec{S}$ is a surface S along with a (consistent) specification of "counterclockwise" at each point:

In $\mathbb{R}^3$, using the "right hand rule", this is equivalent to specifying a unit normal vector N at each point:



The specification of N is often easier to work with.

Any parametric surface $\mathbf{r}: D \xrightarrow{1} S$ is naturally oriented: counterclockwise on $D \subseteq \mathbb{R}^2_{u,v}$ defines a counterclockwise on S:



Working in $\mathbb{R}^3$, the resulting unit normal vector on S at $\mathbf{r}(u,v)$ is

$$N = \boxed{\frac{\mathbf{r}_u \times \mathbf{r}_v}{\|\mathbf{r}_u \times \mathbf{r}_v\|}}$$, which is easier to work with.

If $\vec{S}$ is given, and we find a once-parametrization $\mathbf{r}: D \xrightarrow{1} S$ which also defines the given orientation on S, I'll write $\boxed{\mathbf{r}: \underrightarrow{D} \xrightarrow{1} \vec{S}}$