"Concept Index" for 16.5-9 (WHOAH.) [Andrew Critch Math 53]

All scalar/vector fields assumed C^2 where defined; all integrals assumed proper (no infinities), all boundaries assumed piecewise C^2-parametrizable.

- The de Rham Sequence in $\mathbb{R}^3$:

$$\begin{Bmatrix}\text{scalar}\\ \text{fields}\end{Bmatrix} \xrightarrow[\nabla]{\text{grad}} \begin{Bmatrix}\text{vector}\\ \text{fields}\end{Bmatrix} \xrightarrow[\nabla\times]{\text{curl}} \begin{Bmatrix}\text{vector}\\ \text{fields}\end{Bmatrix} \xrightarrow[\nabla\cdot]{\text{div}} \begin{Bmatrix}\text{scalar}\\ \text{fields}\end{Bmatrix}$$

$$\boxed{\begin{aligned}\mathbb{F} = \text{grad(something)} &\Rightarrow \text{curl}(\mathbb{F}) = 0\\ \mathbb{F} = \text{curl(something)} &\Rightarrow \text{div}(\mathbb{F}) = 0\end{aligned}}$$

→ [p. 1063 $\boxed{3}$ p. 1065 $\boxed{11}$]

- The Poincaré Lemma in $\mathbb{R}^3$:

If dom($\mathbb{F}$) is considered 1-connected (e.g. all of $\mathbb{R}^3$),

$$\boxed{\text{curl}(\mathbb{F}) = 0 \Rightarrow \mathbb{F} = \text{grad(something)}}$$

→ [partially explained p. 1063 $\boxed{4}$]

If dom($\mathbb{F}$) is considered 2-connected (e.g. all of $\mathbb{R}^3$),

$$\boxed{\text{div}(\mathbb{F}) = 0 \Rightarrow \mathbb{F} = \text{curl(something)}}$$

[not explained in text]

Special Vector Fields:

- gradient fields = path indep. fields = (loop) conservative fields
  [p. 1048 $\boxed{3}$, $\boxed{4}$]

- curl fields = "patch indep." fields = "bubble-conservative" fields
  [partially explained p. 1096 $\boxed{3}$]

- Surface integrals of scalar fields wrt surface area (in $\mathbb{R}^3$)

  $\mathbb{r} : D \xrightarrow{1} S,\ \mathbb{r}(u,v) = (x(u,v), y(u,v), z(u,v))$

  $$\boxed{\iint_S f \, d\$ = \iint_D f(u,v) \, \|\mathbb{r}_u \times \mathbb{r}_v\| \, dA_{u,v}} \to [\text{P. } 1082\ \boxed{2}]$$

- O.S. flux integrals of vector fields (in $\mathbb{R}^3$)

  $\mathbb{r} : \underline{D} \xrightarrow{1} \underline{S},\ N = \dfrac{\mathbb{r}_u \times \mathbb{r}_v}{\|\mathbb{r}_u \times \mathbb{r}_v\|}$

  $$\boxed{\iint_{\underline{S}} \star \mathbb{F} \cdot = \iint_D \mathbb{F}(u,v) \cdot (\mathbb{r}_u \times \mathbb{r}_v) \, dA_{u,v} \left(= \iint_S \mathbb{F} \cdot N \, d\$ \right)} \to [\text{P. } 1087\ \boxed{9}]$$

- Approximations: $\operatorname{curl}(\mathbb{F}) \cdot (\underline{a} \times \underline{b}) \sim$ Circulation of $\mathbb{F}$ around $\underline{a}, \underline{b}$ at $\underline{x}$.

  $\operatorname{div}(\mathbb{F}) \, \operatorname{vol}(E) \sim$ Flux of $\mathbb{F}$ out of E. ($\underline{x} \in E$)

- Stokes' (Curl) Theorem:

  $$\boxed{\int_{\partial \underline{S}} \mathbb{F} \cdot = \iint_{\underline{S}} \star \operatorname{Curl}(\mathbb{F}) \cdot}$$

  "Circulation = net curl flux"

  [P. 1093 □]

- Gauss' (Divergence) Theorem:

  $$\boxed{\iint_{\partial E} \star \mathbb{F} \cdot = \iiint_E \operatorname{div}(\mathbb{F}) \, dV}$$

  "outflux = net divergence"

  [P. 1099 □]