"Concept Index" for 16.1–16.4 (Whoah.)

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Math 53

All fields are assumed to be C^2 where defined

- **Curve integrals of Scalar Fields wrt curve length.**

$\mathbb{r}: I \xrightarrow{1} C \subseteq \mathbb{R}^n$, $\mathbb{r}(t) = (x(t), y(t), \ldots)$

$$\boxed{\int_C f\,ds = \int_I f(t)\,|\mathbb{r}_t|\,dt, \quad \underline{\underline{\text{or}}} \qquad \int_C f\sqrt{(dx)^2+(dy)^2} = \int_I f(t)\sqrt{x_t^2+y_t^2}\,dt}$$

→ [p. 1034 $\boxed{3}$]

- **O.C. integrals of (co)vector Fields,** $\mathbb{r}: \underset{\rightarrow}{I} \xrightarrow{1} \underset{\rightarrow}{C} \subseteq \mathbb{R}^n$, $T = \dfrac{\mathbb{r}_t}{|\mathbb{r}_t|}$

$$\boxed{\int_{\underset{\rightarrow}{C}} \mathbb{F}\cdot = \int_I \mathbb{F}(t)\cdot\mathbb{r}_t\,dt = \int_C \mathbb{F}\cdot T\,ds, \quad \underline{\underline{\text{or}}} \qquad \int_{\underset{\rightarrow}{C}} P\,dx + Q\,dy + R\,dz = \int_I \left(P(t)x_t + Q(t)y_t + R(t)z_t\right)dt}$$

→ [p. 1037 $\boxed{7}$, p. 1042 $\boxed{13}$, p. 1043 $\boxed{}$]

- **The Fundamental Theorem of O.C. Integrals**

$$\boxed{\int_{\underset{\rightarrow}{C}} \nabla f\cdot = \int_{\underset{\rightarrow}{C}} df = \Big|_{\text{beg}\,\underset{\rightarrow}{C}}^{\text{end}\,\underset{\rightarrow}{C}} f}$$

→ [p. 1046 $\boxed{2}$]

- **Gradient V.F.'s = Conservative V.F.'s = Path independent V.F.'s**

$\mathbb{F} = \nabla(\text{something}) \Leftrightarrow \displaystyle\int_{\underset{\rightarrow}{\text{loops}}} \mathbb{F}\cdot = 0 \Leftrightarrow \int_{\underset{\rightarrow}{C}} \mathbb{F}\cdot$ depends only on beg/end of $\underset{\rightarrow}{C}$

→ [p. 1048 $\boxed{3}$, p. 1048 $\boxed{4}$]

- The De Rham Sequence in $\mathbb{R}^2$ $\left(\text{scurl}\langle P,Q\rangle = Q_x - P_y\right)$

$$\left\{\begin{matrix}\text{Scalar}\\ \text{Fields}\end{matrix}\right\} \xrightarrow[\nabla]{\text{grad}} \left\{\begin{matrix}\text{Vector}\\ \text{Fields}\end{matrix}\right\} \xrightarrow{\text{scurl}} \left\{\begin{matrix}\text{Scalar}\\ \text{Fields}\end{matrix}\right\}$$

$$\boxed{\underbrace{\mathbb{F} = \nabla(\text{something})}_{\text{“}\mathbb{F}\text{ is conservative”}} \Rightarrow \text{scurl}(\mathbb{F}) = 0}$$

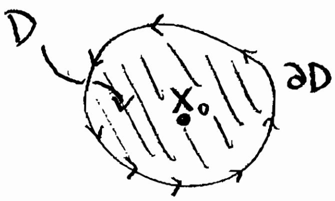
→ [p. 1049 $\boxed{5}$]

- The Poincaré Lemma: <u>If</u> dom($\mathbb{F}$) is considered 1-connected,

$$\boxed{\text{scurl}(\mathbb{F}) = 0 \Rightarrow \mathbb{F} = \underbrace{\nabla(\text{something})}_{\text{“}\mathbb{F}\text{ is conservative”}}}$$

→ [p. 1050 $\boxed{6}$]

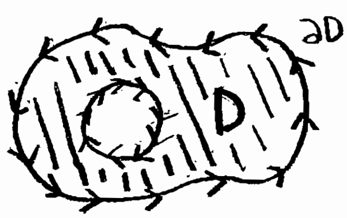
- Scurl as circulation per area at a point:



$$\boxed{\begin{aligned}\int_{\partial D} \mathbb{F}\cdot &= \text{“Circulation of } \mathbb{F} \text{ around } D\text{”}\\ &\approx \text{scurl}(\mathbb{F})(\underset{\bullet}{X_0}) \cdot \text{area}(D)\end{aligned}}$$

as D shrinks around $\underset{\bullet}{X_0}$

- Green's (Scurl) Theorem



$$\boxed{\begin{gathered}\iint_D \text{scurl}(\mathbb{F})\,dA = \int_{\partial D} \mathbb{F}\cdot \;,\ \underline{\underline{\text{or}}}\\[1em] \iint_D (Q_x - P_y)\,dA = \int_{\partial D} P\,dx + Q\,dy\end{gathered}}$$

Need:

- $\mathbb{F}$ is C^1 on D
- ∂D is C^1

" Sum of tiny circulations = total circulation "